LESSON 3

6.RP.2, 6.RP.3.b

Solving Unit Rate Problems

1 GETTING THE IDEA

A **rate** is a comparison, or ratio, of two quantities with different units. For example, a store sells 3 T-shirts for $15. The comparison $15 to 3 T-shirts is a rate. This rate can also be written as $\frac{\$15}{3 \text{ T-shirts}}$.

When a rate compares a quantity to one unit of another quantity, the rate is a **unit rate**.

The rate $\frac{\$15}{3 \text{ T-shirts}}$ is *not* a unit rate because the rate compares the cost to more than one T-shirt. In a unit rate, the second quantity (or denominator) should be 1 unit.

A speed limit is a unit rate. For example, the speed limit of 55 miles per hour compares the distance to 1 hour. A speed of 55 miles per hour is equivalent to traveling 55 miles in each 1-hour time span. This rate can also be written as $\frac{55 \text{ miles}}{1 \text{ hour}}$.

$\frac{\$15}{3 \text{ T-shirts}}$ **not** a unit rate

$\frac{55 \text{ miles}}{1 \text{ hour}}$ unit rate

Every rate can be written as a unit rate.

Example 1

A store sells 3 T-shirts for $15. What is the cost per T-shirt?

Strategy **Use a diagram to find the unit rate.**

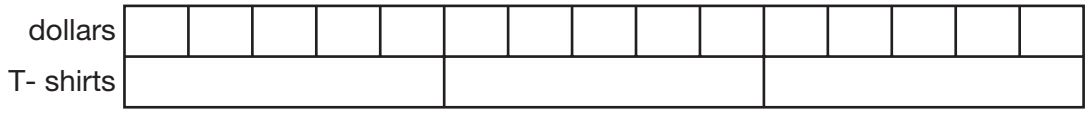
Step 1 Write the given rate as a fraction.

Because the question asks for the cost per T-shirt, the rate compares dollars to T-shirts.

$\frac{\$15}{3 \text{ T-shirts}}$

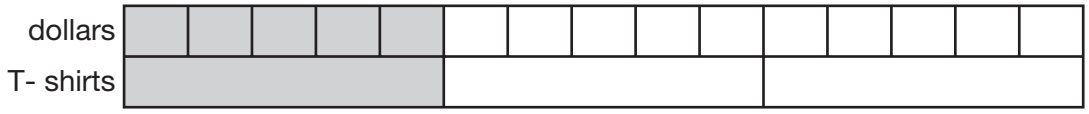
Step 2 Draw a diagram to represent the rate.

There are 15 dollars, so divide the top rectangle into 15 equal parts. There are 3 T-shirts, so divide the bottom rectangle into 3 equal parts.



Step 3 Use the diagram to find the unit rate.

The unit rate is the rate that compares the cost to one T-shirt.



There are five dollars for one T-shirt. So, the unit rate is $\frac{\$5}{1 \text{ T-shirt}}$, or \$5 per T-shirt.

Solution **The cost per T-shirt is \$5.**

Example 2

Changying bikes 36 miles in 3 hours. How many miles per hour does she travel?

Strategy **Use an equation.**

Step 1 Write a rate that compares distance to time.

$$\frac{\text{Distance}}{\text{time}} = \frac{36 \text{ miles}}{3 \text{ hours}}$$

Step 2 Write an equation.

The unit rate is the number of miles per 1 hour. The given rate and the unit rate are equivalent.

$$\frac{36 \text{ miles}}{3 \text{ hours}} = \frac{? \text{ miles}}{1 \text{ hour}}$$

Step 3 Use division to find the missing number.

Because 3 hours ÷ 3 = 1 hour, write an equivalent rate by dividing 36 miles by 3:

$$\frac{36 \text{ miles} \div \mathbf{3}}{3 \text{ hours} \div \mathbf{3}} = \frac{12 \text{ miles}}{1 \text{ hour}}$$

Solution **Changying travels $\frac{12 \text{ miles}}{1 \text{ hour}}$, or 12 miles per hour.**

Note that in both Example 1 and Example 2, the unit rate can be found by dividing the numerator by the denominator and simplifying the fraction. The unit rate for $\frac{\$15}{3 \text{ T-shirts}}$ is 15 ÷ 3 = 5 dollars. The unit rate for $\frac{36 \text{ miles}}{3 \text{ hours}}$ is 36 ÷ 3 = 12 miles per hour. This is because the quotient can be written as a fraction over a denominator of 1.

To find a unit rate, write the ratio as a fraction and divide the numerator by the denominator.

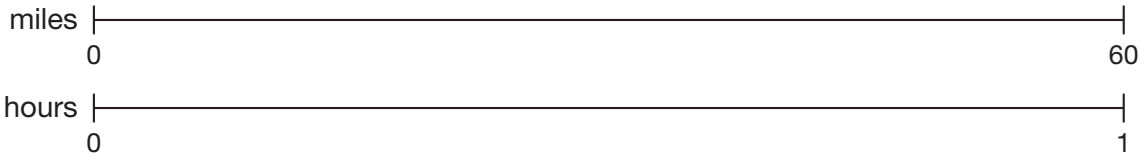
Example 3

Ramon drives at a rate of 60 miles per hour. He stops for lunch after driving for $\frac{1}{2}$ hour. How many miles did Ramon drive in $\frac{1}{2}$ hour?

Strategy **Use a diagram to find the distance.**

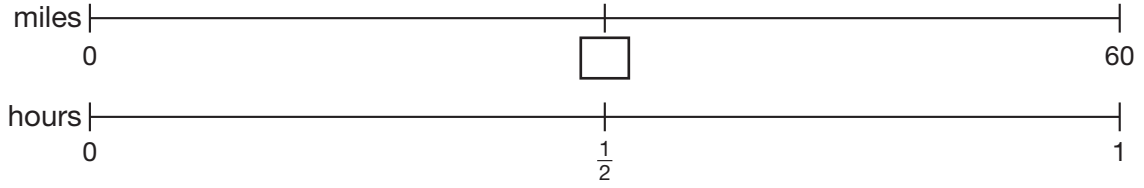
Step 1 Draw a diagram to show the given unit rate.

The given unit rate is $\frac{60 \text{ miles}}{1 \text{ hour}}$.



Step 2 Label the known and unknown quantities on the diagram.

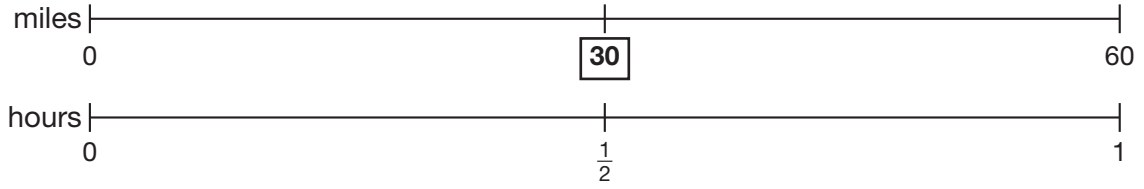
Ramon drove for $\frac{1}{2}$ hour. Show $\frac{1}{2}$ on the number line for hours.



Step 3 Use the diagram to find the unknown quantity.

The tick mark divides 60 into two equal parts.

Since 60 ÷ 2 = 30, each part must represent 30.



Solution **In $\frac{1}{2}$ hour, Ramon traveled 30 miles.**

Example 4

Eva pays a company $30 every 6 months to host her Web site.

(a) How much does Eva pay per month?

(b) How much does Eva pay for 9 months?

Strategy **Use equations to find the equivalent rates.**

Step 1 Write the given rate.

$$\frac{\text{Cost}}{\text{Time}} = \frac{\$30}{6 \text{ months}}$$

Step 2 Use division to find the cost per month.

Divide 30 by 6 to simplify the fraction and find the unit rate.

$$\frac{30}{6} = 30 \div 6 = 5$$

This is equal to $\frac{5}{1}$, or $\frac{\$5}{1 \text{ month}}$.

The unit rate is $5 per month.

Step 3 Write an equation that can be used to answer part b.

Part b asks for the cost for 9 months. Use the unit rate to write an equation.

$$\frac{\$5}{1 \text{ month}} = \frac{?}{9 \text{ months}}$$

Step 4 Use multiplication to find the missing number.

Because 1 month $\times$ 9 = 9 months, write an equivalent rate by multiplying the numerator and denominator by 9:

$$\frac{\$5 \times 9}{1 \text{ month} \times 9} = \frac{\$45}{9 \text{ months}}$$

Solution **Eva pays $5 per month and $45 for 9 months.**

2 COACHED EXAMPLE

A water pump can pump 250 gallons from a pool in 5 minutes.

(a) What is the unit rate of the pump in gallons per minute?

(b) How long will it take to pump 450 gallons from the pool?

(a) The unit rate compares _____ to _____.

The given rate is _____ gallons in _____ minutes.

Write the given rate as a fraction.

$$\frac{\square \text{ gallons}}{\square \text{ minutes}}$$

Divide the numerator by the denominator to find the unit rate.

_____ ÷ _____ = _____

The unit rate is _____.

(b) Use the _____ rate to write an equation to solve the problem.

$$\frac{\square \text{ gallons}}{1 \text{ minute}} = \frac{450 \text{ gallons}}{? \text{ minutes}}$$

Multiply the numerator and denominator by _____ to find the equivalent fraction.

$$\frac{\square \times \square \text{ gallons}}{1 \times \square \text{ minutes}} = \frac{450 \text{ gallons}}{\square \text{ minutes}}$$

It takes _____ minutes to pump 450 gallons from the pool.

The unit rate of the pump is _____ gallons per minute. It takes _____ minutes to pump 450 gallons of water from the pool.

3 LESSON PRACTICE

1 Four muffins cost $12. Complete the model to find the cost per muffin.

dollars	
muffins	

The cost per muffin is _____.

2 Select a rate that is a unit rate. Circle all that apply.

A. $\frac{36 \text{ cookies}}{3 \text{ batches}}$

B. 60 gallons every 4 hours

C. $40 per ticket

D. $\frac{10 \text{ miles}}{10 \text{ hours}}$

E. $\frac{1}{2}$ cup per serving

F. 15 people for every row

G. $\frac{12 \text{ roses}}{1 \text{ vase}}$

3 Draw a line from the rate to its unit rate.

A. $\frac{225 \text{ feet}}{5 \text{ minutes}}$	•	• 18 feet per minute
B. $\frac{450 \text{ feet}}{9 \text{ minutes}}$	•	• 30 feet per minute
C. $\frac{36 \text{ feet}}{2 \text{ minutes}}$	•	• 45 feet per minute
D. $\frac{210 \text{ feet}}{7 \text{ minutes}}$	•	• 50 feet per minute

4 A line-painting truck is painting a line on the side of a roadway. The truck paints at a rate of 8 miles per hour. How many miles can be painted in $\frac{1}{4}$ hour?

_____ miles can be painted in $\frac{1}{4}$ hour.

5 Twelve boxes of pencils cost $16. Which equation could you use to find the cost per box?

A. $\frac{12 \text{ boxes}}{\$16} = \frac{\square \text{ boxes}}{\$1}$

B. $\frac{\$16}{12 \text{ boxes}} = \frac{\$1}{\square \text{ boxes}}$

C. $\frac{12 \text{ boxes}}{\$16} = \frac{\$\square}{1 \text{ box}}$

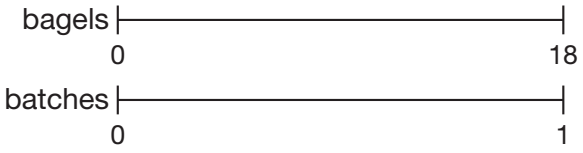
D. $\frac{\$16}{12 \text{ boxes}} = \frac{\$\square}{1 \text{ box}}$

6 Cassie drives 165 miles in 3 hours. Circle the numbers that make the rates below equivalent to the rate 165 miles in 3 hours.

Cassie drives 55 / 60 / 65 miles per hour.

Cassie drives 275 / 300 / 325 miles in 5 hours.

7 Troy's recipe for bagels makes 18 bagels per batch. Troy makes $\frac{2}{3}$ batch of bagels. How many bagels does Troy make? Complete the model to solve the problem.



Troy makes _____ bagels.

8 Are the rates equivalent? Select True or False for each equation.

A. 35 feet for every 7 seconds = 7 feet for every second ○ True ○ False

B. $\frac{120 \text{ cans}}{5 \text{ cases}} = \frac{25 \text{ cans}}{1 \text{ case}}$ ○ True ○ False

C. \$12 per hour = \$48 for every 4 hours ○ True ○ False

D. $\frac{1 \text{ teacher}}{15 \text{ students}} = \frac{6 \text{ teachers}}{90 \text{ students}}$ ○ True ○ False

9 Takashi ran 200 yards in 40 seconds. Ana ran 150 yards in 25 seconds.

Part A

Can you tell who runs faster without rewriting the rates? Explain why or why not.

Part B

Who ran faster? Show your work.

10 Use numbers from the box to complete the table so that the rates are equivalent.

Dollars		90	180	
Hours	1	5		12

10
15
18
45
216
360

11 The speed limit is 50 miles per hour. Kyle is driving to a baseball game that starts in 2 hours. Kyle is 130 miles away from the baseball field. If Kyle drives at the speed limit, will he arrive in time? Show your work.

12 A new issue of Padma's favorite magazine comes out every month. If she buys the magazine at the bookstore, the cost is $5 per month. If she subscribes, the cost is $57 per year. Should Padma buy each issue separately or should she get a subscription? Show your work.